Phase study of oscillatory resistances in high mobility GaAs/AlGaAs devices: Indications of a new class of integral quantum Hall effect

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An experimental study of the high mobility GaAs/AlGaAs system at large-ν indicates several distinct phase relations between the oscillatory diagonal- and Hall- resistances, and suggests a new class of integral quantum Hall effect, which is characterized by "anti-phase" Hall- and diagonal-resistance oscillations.

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A 2-dimensional electron system (2DES) at high magnetic fields, B, and low temperatures, T, exhibits the integral quantum Hall effect (IQHE), which is characterized by plateaus in the Hall resistance R_{xy} vs. B, at $R_{xy} = h/ie^2$, with i = 1,2,3,... and concurrent vanishing diagonal resistance R_{xx} as $T \rightarrow 0$ K, in the vicinity of integral filling factors of Landau levels, i.e., $\nu \approx i$.[1, 2, 3] With the increase of the electron mobility, μ, at a given electron density, n, and T, IQHE plateaus typically become narrower as fractional quantum Hall effects (FQHE) appear in the vicinity of $\nu \approx p/q$, at $R_{xy} = h/[(p/q)e^2]$, where p/q denotes mostly odd-denominator rational fractions.[2, 3] Experimental studies of the highest mobility specimens have mostly focused upon FQHE and other novel phases.[2, 3, 4, 5] Meanwhile, the possibility of new variations of IQHE that might appear with the canonical effect in the reduced-disorder specimen, especially at large-ν, has been largely unanticipated by experiment. Here, we show that three distinct phase relationships can occur between the oscillatory diagonal- and Hall- resistances in the high-mobility specimen at $\nu > 5$, and that IQHE can be manifested in two of these variations. The results therefore identify one new class of IQHE, as they provide insight into the origin of oscillatory variations in the Hall effect, and their evolution into plateaus, in the low-B large-ν regime of the radiation-induced zero-resistance states in the *photoexcited* high mobility 2DES.[6, 7, 8] Thus, these experiments also serve to characterize, classify, and clarify the large-ν dark properties. The results recall previous suggestions of new phenomena in the $\nu >> 1$ limit.[9]

Figure 1(a) exhibits data from a low mobility Hall bar specimen with $n = 3.2 \times 10^{11} cm^{-2}$ and $\mu = 0.4 \times 10^6 cm^2/V-s$. Here, large amplitude Shubnikov-de Haas (SdH) oscillations in R_{xx} lead into zero-resistance states with increasing B, as R_{xy} exhibits plateaus at $R_{xy} = h/ie^2$ for $\nu \approx i$, with i = 2,4,6,... This canonical low-mobility IQHE system is known to exhibit a resistivity/resistance rule,[10, 11] at a each T,[12] whereby $R_{xx} \propto B[dR_{xy}/dB]$ and dR_{xy}/dB is the B-field derivative of R_{xy}.[10] Indeed, a comparison of R_{xx} (Fig. 1(a)) and $B[dR_{xy}/dB]$ (Fig. 1(b)) confirms agreement.[10, 11, 12, 13]

For the sake of further analysis, Fig. 1(c) shows the oscillatory part of the diagonal (ΔR_{xx}) and Hall (ΔR_{xy}) resistances vs. ν. Here, $\Delta R_{xy} = R_{xy} - R_{xy}^{back}$ and $\Delta R_{xx} = R_{xx} - R_{xx}^{back}$, as R_{xy}^{back} and R_{xx}^{back} are the background resistances shown in Fig. 1(a). As evident from Fig. 1(c), the quantum Hall characteristics of Fig. 1(a) yield approximately orthogonal oscillations in ΔR_{xx} and ΔR_{xy} such that $\Delta R_{xx} \approx -cos(2\pi[\nu/2])$

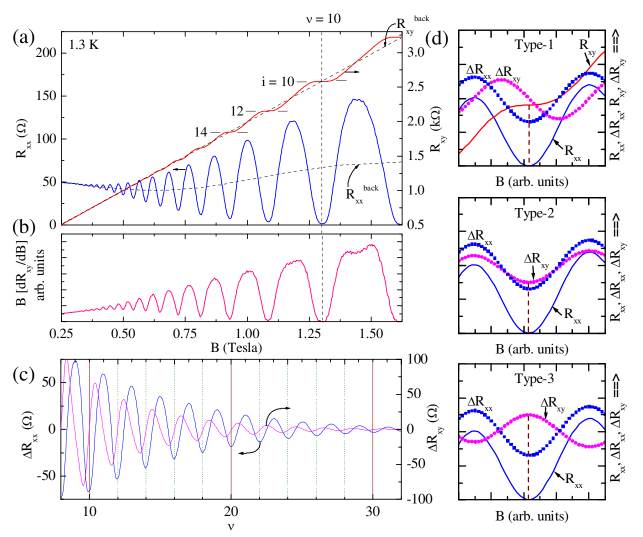

FIG. 1: (a) Integral quantum Hall plateaus, at $R_{xy} = h/ie^2$, coincide with resistance minima in R_{xx} in a GaAs/AlGaAs Hall bar device. (b) A comparison of $B[dR_{xy}/dB]$ shown here with R_{xx} in Fig. 1(a) suggests that $R_{xx} \propto B[dRxy/dB]$, as per the resistivity rule (ref. 10). (c) A quantum Hall system at large-ν also exhibits an approximately 90^0 phase shift between the oscillatory parts of R_{xx} and R_{xy}. (d) This panel illustrates the three types of magnetoresistance oscillations that are reported in this study.

and $\Delta R_{xy} \approx sin(2\pi[\nu/2])$. We denote the quantum Hall characteristics of Fig. 1(a)-(c) as "Type-1" characteristics, and sketch the essentials at the top of Fig. 1(d).

This study reports on other observable phase relations in the high mobility 2DES. We find, for instance, a "Type-2" case, where $|R_{xy}|$ is enhanced at the R_{xx} oscillation peaks and the ΔR_{xy} oscillations are in-phase with the R_{xx} or ΔR_{xx} oscillations, as illustrated in Fig. 1(d) [center]. There also occurs the "Type-3" case where $|R_{xy}|$ is suppressed at the R_{xx} oscillation maxima and the ΔR_{xy} oscillations are phase-shifted by "π" with respect to the R_{xx} or ΔR_{xx} oscillations, as shown in Fig. 1(d)[bottom]. Here, we survey these experimentally observed phase relations and related crossovers in the high mobility 2DES, and then focus on the "Type-3" case, which also brings with it, remarkably, IQHE.

Simultaneous lock-in based electrical measurements of the diagonal resistance and the Hall effect were carried out at $T > 0.45K$, with matched lock-in time constants, and sufficiently slow B-field sweep rates. The B-field was calibrated by ESR of DPPH.[6] We are able to rule out spurious contributions in the phase shift, between ΔR_{xx} and ΔR_{xy}, originating from the B-sweep rate, the data acquisition rate, lock-in integration time, and other typical experimental variables. As usual, the observed magnetoresistance oscillations became weaker at higher T, and few oscillations were evident for $\nu > 20$ at $T > 1.7K$. Thus, this study focused upon $0.45 < T < 1.7K$, where T-induced changes in the phase relations were not discerned. The observed phase relations also did not show an obvious dependence on the sample geometry. We note, parenthetically, that the phase relation between the oscillatory Hall- and diagonal- resistances could often be identified by the trained eye. Thus, background subtraction helps mainly to bring out the Hall oscillations from R_{xy}, and make possible ΔR_{xx}, ΔR_{xy} overlays, for phase comparison. Typically, R_{xx}^{back} followed either the midpoints (e.g. Fig. 1(a)) or the minima (e.g. Fig. 2(d)) of the R_{xx} oscillations. For R_{xy}^{back}, since $|R_{xy}^{back}| >> |\Delta R_{xy}|$, we used a two pass procedure where the first pass identified $\approx 99\%$ of R_{xy}^{back} through a linear-fit of R_{xy}, and a spline fit in the second pass then accounted for the $\approx 1\%$ residual term. Here, at the second pass, R_{xy}^{back} was optimized to make possible ΔR_{xx}, ΔR_{xy} overlays. Finally, although μ has been provided, μ alone seems to be insufficient for classifying the observed phenomena in high-μ specimens. Here, the high mobility condition was realized in GaAs/AlGaAs by brief illumination with a red LED.

For a high mobility square shaped specimen with $n = 3 \times 10^{11} cm^{-2}$ and $\mu = 1.1 \times 10^7 cm^2/Vs$ that shows IQHE up to $i \approx 40$, Fig. 2(a) illustrates R_{xx}, R_{xy}, and ΔR_{xy} for both B-directions, using the convention $R_{xy} > 0$ for $B > 0$. Figures 2(b) and (c) confirm similar behavior for both B-directions once the anti-symmetry in ΔR_{xy} under B-reversal is taken into account. Fig. 2(b) indicates

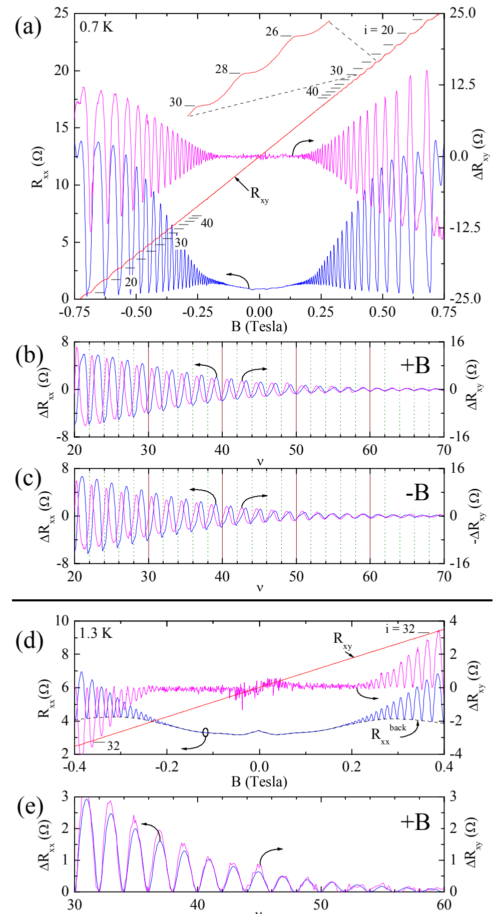

FIG. 2: (a) R_{xx}, R_{xy}, and the oscillatory Hall resistance, ΔR_{xy}, are shown over low magnetic fields, B for a high mobility square shape GaAs/AlGaAs specimen. Here, $\Delta R_{xy}(-B) = -\Delta R_{xy}(+B)$. (b) The oscillatory diagonal resistance (ΔR_{xx}) and ΔR_{xy} have been plotted vs. ν to compare their relative phases for $+B$. For $20 \leq \nu < 46$, ΔR_{xx} and ΔR_{xy} are approximately orthogonal as in Fig. 1(c). For $56 \leq \nu \leq 70$, ΔR_{xx} and ΔR_{xy} are approximately in-phase, unlike at $\nu < 46$. (c) As above for $-B$. Note that the right ordinates in Fig. 2(b) and Fig. 2(c) show $+\Delta R_{xy}$ and $-\Delta R_{xy}$, respectively, in order to account for the antisymmetry in ΔR_{xy} under B-reversal. (d) R_{xx}, R_{xy}, and ΔR_{xy} have been shown for a Hall bar specimen with $n = 2.9 \times 10^{11} cm^{-2}$ and $\mu = 6 \times 10^6 cm^2/Vs$, which exhibits in-phase R_{xx} and ΔR_{xy} oscillations. Note the absence of discernable Hall plateaus in R_{xy}. (e) Here, the amplitudes of ΔR_{xx} and ΔR_{xy} show similar ν -variation.

that from $20 \leq \nu < 46$, ΔR_{xy} oscillations are approximately orthogonal to the ΔR_{xx} oscillations, as in Fig. 1(c). This feature, and the manifestation of Hall plateaus in Fig. 2(a), confirms that the IQHE observed here is the canonical effect. A remarkable and interesting feature in Fig. 2(b) is that, following a smooth crossover, ΔR_{xx} and ΔR_{xy} become in-phase, i.e., "Type-2", for $\nu \geq 56$. That is, with increasing B (or decreasing ν), the system

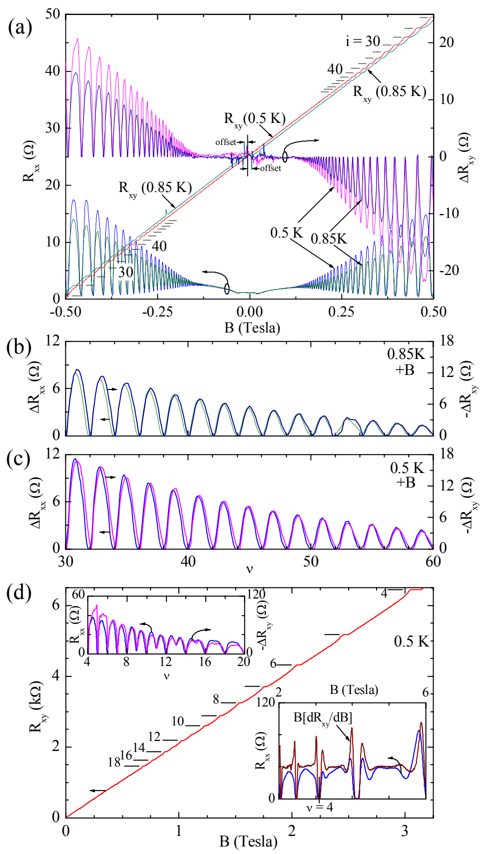

FIG. 3: (a) Data for a square shape GaAs/AlGaAs specimen, where the magnitude of R_{xy} is suppressed at the R_{xx} oscillation maxima. Here, the $+B$ and the $-B$ portions of the $R_{xy}(0.85K)$ curve have been offset in opposite directions along the abscissa with respect to the $R_{xy}(0.5K)$ curve, for the sake of presentation. Note the well-developed plateaus in R_{xy}. (b) At $T = 0.85K$, a plot of ΔR_{xx} and $-\Delta R_{xy}$ confirms a Type-3 phase relation. (c) Same as (b) but at $T = 0.5K$. (d) The main panel shows the Hall resistance vs. B, with relatively narrow Hall plateaus, down to around filling factor $\nu = 4$. Left Inset: As in Fig. 3(b) and 3(c) above, $-\Delta R_{xy}$ follows R_{xx}, down to nearly $\nu = 5$. Right Inset: For $\nu \leq 4$, a better correspondence developed between R_{xx} and $B[dR_{xy}/dB]$.

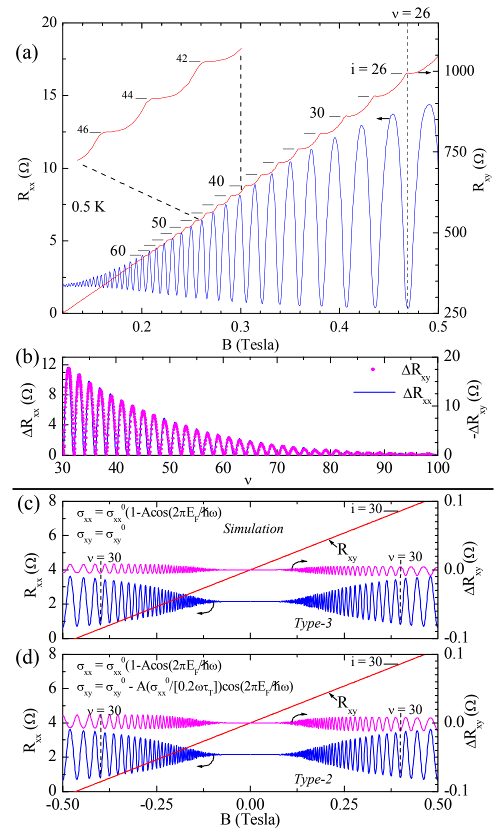

FIG. 4: (a) R_{xx} and R_{xy} in a high mobility GaAs/AlGaAs specimen that exhibits deep R_{xx} minima and even-integral Hall plateaus at $R_{xy} = h/ie^2$. (b) Here, $-\Delta R_{xy}$ follows ΔR_{xx}, indicative of a π phaseshift and a "Type-3" relationship, unlike the typical quantum Hall situation, see Fig. 1(c). (c) Simulations suggest that oscillatory scattering corrections to the diagonal conductivity alone can produce Type-3 oscillations, via the tensor relations for the resistivities. (d) Simulations of a semiempirical model that includes both a scattering contribution in σ_{xy}, and a reduction in the relaxation time with respect to the transport lifetime, indicate the possibility also of Type-2 oscillations, with $|\Delta R_{xy}| << |\Delta R_{xx}|$. A comparison of Fig. 4(c) and 4(d) helps to convey the τ-induced Type-3 to Type-2 transformation.

exhibits a Type-2 $\rightarrow$ Type-1 transformation.

Figs. 2(d) and 2(e) provide further evidence for in-phase Type-2 oscillations in a Hall bar. Here, the Hall oscillations tend to enhance the magnitude of R_{xy} at the R_{xx} SdH maxima ("Type-2"), even as Hall plateaus are imperceptible in the R_{xy} curve. Yet, from Fig. 2(d), it is clear that ΔR_{xy} is a Hall effect component, and not a misalignment offset admixture of R_{xx} into R_{xy}, since ΔR_{xy} is antisymmetric under B-reversal.

Figure 3 illustrates the third ("Type-3") phase relation in a high mobility square shape 2DES with $n = 2.9 \times 10^{11} cm^2$ and $\mu = 1 \times 10^7 cm^2/V-s$. Although μ for this specimen is similar to the one examined in Fig. 2(a)-(c), the experimental results do look different. Figure 3(a) exhibits data taken at $T = 0.85K$ and $T = 0.5K$. Fig. 3(a) shows that the main effect of changing T is to modify the amplitude of the ΔR_{xx} and ΔR_{xy} oscillations, so that oscillatory effects persist to a lower B at the lower T. The data of Fig. 3(a) also show that ΔR_{xy} tends to reduce the magnitude of R_{xy} over the B-intervals corresponding to the R_{xx} peaks, as in Fig. 1d(bottom), the Type-3 case. Meanwhile, quantum Hall plateaus are easily perceptible in R_{xy}, see Fig 3(a). Figures 3(b) and 3(c) demonstrate that for $+B$, for example, ΔR_{xx} and $-\Delta R_{xy}$ show the same lineshape for $30 \leq \nu \leq 60$, and the phase relation does not change with T. Indeed, this correlation held true down to nearly $\nu = 5$, see left inset of Fig. 3(d),

as narrow IQHE plateaus were manifested in R_{xy}, see Fig. 3(d). For $\nu \leq 4$, however, R_{xx} correlated better with $B[dR_{xy}/dB]$, see right inset of Fig. 3(d), than with $-\Delta R_{xy}$, which suggested that the resistivity/resistance rule[10] comes into play at especially low-ν here, as the system undergoes a Type-3 $\rightarrow$ Type-1 transformation, with decreasing ν.

An expanded data plot of Type-3 transport is provided in Fig. 4(a). This plot shows plateaus in R_{xy} and deep minima in R_{xx} to very low-B, as quantum Hall plateaus in R_{xy} follow $R_{xy} = h/ie^2$, to an experimental uncertainty of ≈ 1 percent. Although the IQHE data of Fig. 4(a) again appear normal at first sight, the remarkable difference becomes apparent when ΔR_{xx} and -ΔR_{xy} are plotted vs. ν, as in Fig. 4(b). Here, we find once again a phase-shift of "π" ("Type-3") between ΔR_{xx} and ΔR_{xy} (Fig. 4(b)), as in Fig. 3(b) and (c), that is distinct from the canonical ("Type-1") phase relationship exhibited in Fig. 1(c). In Fig. 4(a), the reported phase relation can even be discerned by the trained eye.

It is possible to extract some insight from the phase relationships observed here between R_{xx} (or ΔR_{xx}) and ΔR_{xy}. The Type-1 orthogonal phase relation of Fig. 1(c) can be viewed as a restatement of the empirical resistivity/resistance rule, since the data of Fig. 1(a) yield both Fig. 1(b) and Fig. 1(c). Theory suggests that this rule might follow when R_{xx} is only weakly dependent on the local diagonal resistivity ρ_{xx} and approximately proportional to the magnitude of fluctuations in the off-diagonal resistivity ρ_{xy}, when ρ_{xx} and ρ_{xy} are functions of the position.[14] Thus, specimens exhibiting the resistivity rule (and Type-1 oscillations) seem likely to reflect density fluctuations/inhomogeneities.[14]

For Type-2 and Type-3 oscillations, note that the specimens of Figs. 2 - 4 satisfy $\omega\tau_T > 1$, with ω the cyclotron frequency, and τ_T the transport lifetime, at $B > 0.001$ (or 0.002) T. Yet, one might semi-empirically introduce oscillations into the diagonal conductivity, σ_{xx}, as $\sigma_{xx} = \sigma^0_{xx}(1 - Acos(2\pi E_F/\hbar\omega))$.[15, 16, 17] Here, the minus sign ensures the proper phase, while $\sigma^0_{xx} = \sigma_0/(1 + (\omega\tau_T)^2)$, σ_0 is the dc conductivity, E_F is the Fermi energy, and $A = 4c[(\omega\tau_T)^2/(1 + (\omega\tau_T)^2)][X/sinh(X)]exp(-\pi/\omega\tau_S)$, where $X = [2\pi^2 k_B T/\hbar\omega]$, τ_S is single particle lifetime, and c is of order unity.[15, 17, 18] Simulations with σ_{xx} as given above, and $\sigma_{xy} = \sigma^0_{xy} = -(\omega\tau_T)\sigma^0_{xx}$, indicate oscillations in both R_{xx} and R_{xy} via $\rho_{xx} = \sigma_{xx}/(\sigma^2_{xx} + \sigma^2_{xy})$ and $\rho_{xy} = \sigma_{xy}/(\sigma^2_{xx} + \sigma^2_{xy})$, and a Type-3 phase relationship, see Fig. 4(c), with $|\Delta R_{xy}| << |\Delta R_{xx}|$. That is, an oscillatory σ_{xx} can also lead to R_{xy} oscillations.

As a next step, one might introduce an oscillatory $\sigma_{xy} = \sigma^0_{xy}(1 + Gcos(2\pi E_F/\hbar\omega))$, where $G = 2c[(1 + 3(\omega\tau_T)^2)/((\omega\tau_T)^2(1 + (\omega\tau_T)^2))][X/sinh(X)]exp(-\pi/\omega\tau_S)$.[16, 17] Upon inverting the tensor including oscillatory σ_{xy} and σ_{xx}, Type-3 oscillations were still obtained, as in Fig. 4(c).

Finally, the strong B-field σ_{xy} follows $\sigma_{xy} = \sigma_{xx}/\omega\tau - ne/B$ in the self-consistent Born approximation for short range scattering potentials, when τ is the relaxation time in the B-field.[16] Although, the dominant scattering mechanism is long-ranged in GaAs/AlGaAs devices, we set $\sigma_{xy} = (\sigma^0_{xx}/\omega\tau - ne/B) - A(\sigma^0_{xx}/\omega\tau)cos(2\pi E_F/\hbar\omega)$. When $\tau = \tau_T$, this approach again yielded Type-3 phase relations, as in the discussion above. Next, we examined the case $\tau < \tau_T$, in order to account for the possibility that τ in a B-field may possibly come to reflect τ_S, which typically satisfies $\tau_S < \tau_T$ for small angle scattering by long-range scattering-potentials.[18] Remarkably, a reduction in τ, which corresponds to changing the nature of the potential landscape, converted Type-3 (phase-shift by π) to Type-2 (in-phase) oscillations, see Fig. 4(c) and 4(d).

If density fluctuations at large length scales produce Type-1 characteristics,[14] and Type-2 oscillations require a difference between τ_T and τ as suggested above, then the observation of Type-1 and Type-2 oscillations in the same measurement (Fig. 2(b) and (c)) is consistent because long length-scale potential fluctuations can produce both modest density variations and a difference between τ_T and τ_S (or τ).[18] Perhaps, with increasing B, there is a crossover from "Type-2" to "Type-1" before R_{xy} plateaus become manifested, and thus, IQHE is not indicated in the Type-2 regime, see Fig. 2 (a) and (d).

Specimens exhibiting Type-3 oscillations and associated IQHE suggest better homogeneity in n, which is confirmed by oscillations to extremely low-B (see Fig. 4(a) and (b)), to nearly $\nu = 100$ at $T = 0.5$ K. The relatively narrow plateaus at high-B (see Fig. 3(d)) hint at a reduced role for disorder-induced localization.[2] In this case, perhaps there are other new mechanisms contributing to the large $|\Delta R_{xy}|$, and plateau formation, in the high-μ system. It could be that new physics serves to create/maintain/enhance the mobility gap or suppress backscattering in the higher Landau level here, which assists in the realization of the "Type-3" characteristics and IQHE to very low B. As theory and experiment have already suggested such possibilities in the higher Landau levels,[4, 5, 9, 10], such ideas seem to merit consideration.[19]

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[19] This manuscript was submitted to Physical Review Letters on 9 March 2006, and it was rejected on 19 June 2006. A prior manuscript was submitted to Nature Physics on 5 December 2005, and it was rejected on 3 March 2006.
Some selected thought-provoking comments by the reviewers:
(i) "Why should there be a "residual term" at all? The R_{xy} background needs to be linear". (The Hall sensor industry would be extremely pleased if there was a Hall system, which did not exhibit any non-linearities in the "background".)
(ii) "The authors were dealing with very high filling factor regime. This means the magnetic field is very small. A tiny error in it would give rise to a big error bar.... On the other hand, the linearity becomes better at high magnetic fields. That the author did not observe any phase shift at high fields may suggest that this poor linearity is the source for the observed phase shift". (Fig. 1c shows a phase shift at "high B".)
We stand by these experimental results, the analysis, and our claim that there occur two classes of IQHE in the high mobility 2DES specimen, one that goes together with Type-1 oscillations, and another (see Figs. 3 and 4) that goes together with Type-3 oscillations.